Determine which characteristics of an algorithm the following procedures they lack.

a) **procedure** *double*(n: positive integer)

while $n > 0$

$n := 2n$

c) **procedure** *sum*(n: positive integer)

$sum := 0$

while $i < 10$

$sum := sum + i$

Describe an algorithm that takes as input a list of n distinct integers and finds the location of the largest even integer in the list or returns 0 if there are no even integers in the list.

Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

procedure *bubble sort*($a_1, a_2, \dots, a_n$: real numbers with $n \geq 2$)
for $i := 1$ **to** $n - 1$
    for $j := 1$ **to** $n - i$
        if $a_j > a_{j+1}$ **then** interchange a_j and a_{j+1}
{$a_1, a_2, \dots, a_n$ is in increasing order}

Use the insertion sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

procedure *insertion sort*($a_1, a_2, \dots, a_n$: real numbers with $n \geq 2$)
for $j := 2$ **to** n
begin
    $i := 1$
    while $a_j > a_i$
        $i := i + 1$
    $m := a_j$
    for $k := 0$ **to** $j - i - 1$
        $a_{j-k} := a_{j-k-1}$
    $a_i := m$
end {$a_1, a_2, \dots, a_n$ are sorted}

Use the definition of “$f(x)$ is $O(g(x))$” to show that $2^x + 17$ is $O(3^x)$.

Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

a) $f(x) = 2x^2 + x^3 \log x$
b) $f(x) = 3x^5 + (\log x)^4$
c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
d) $f(x) = (x^3 + 5\log x)/(x^4 + 1)$

Give a big-*O* estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g)$, use a simple function g of smallest order.

$(n^n + n2^n + 5^n)(n! + 5^n)$

For each of these functions, determine whether that is $\Omega(x)$ and whether it is $\Theta(x)$.

a) $f(x) = 10$
b) $f(x) = 3x + 7$
c) $f(x) = x^2 + x + 1$
d) $f(x) = 5\log x$
e) $f(x) = \lfloor x \rfloor$
f) $f(x) = \lceil x/2 \rceil$

Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x)}$ is $O(2^{g(x)})$?

Show that the greedy algorithm for making change for n cents using quarters, dimes, nickels, and pennies has $O(n)$ complexity measured in terms of comparisons needed.

Prove or disprove that if $a|bc$, where a, b, and c are positive integers, then $a|b$ or $a|c$.

Show that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d, and m are integers with $m \geq 2$, then $a - c \equiv b - d \pmod{m}$.

Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod{8}$.

We call a positive integer perfect if it equals the sum of its positive divisors other than itself

(a) Show that 6 and 28 are perfect.

(b) Show that $2^{p-1}(2^p - 1)$is a perfect number when 2^{p-1} is prime.

If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?